GUIDED NOTES- Completing the Square

Goal: to be able to solve quadratic equations by completing the square.

Completing the square: the process of making a quadratic factorable.

Sometimes a quadratic does not factor. We can make it "factorable" by the process of completing the square. Because we are solving, the equation should be set equal to 0. We should also always check to see if a problem is easily factorable first, to make sure completing the square is truly necessary.

Step 1 – The constant must be on the other side of the equals sign (leave room)
Step 2- (See Next Page #5 & 6)
Step 3 – Find one half of b, the coefficient of x (divide by 2)
Step 4 – Square the result of step 2
Step 5 – Add the result to both sides of the equation
Step 6- (See Next Page #5 & 6)
Step 7 – Factor the left, simplify the right
Step 8 – Take the square root of both sides and solve the equation. ✱ Be careful → square root is + or −

EXAMPLES:

Ex1 $x^2 - 10x + 16 = 0$

$x^2 - 10x + 25 = -16 + 25$

$-10/2 = -5^2$

$x^2 - 10x + 25 = 9$

$(x-5)(x-5) = 9$

$\sqrt{(x-5)^2} = \sqrt{9}$

$x - 5 = 3 \quad x - 5 = -3$

$x = 8 \quad x = 2$

Ex2 $x^2 - 18x + 72 = 0$

$x^2 - 18x + 81 = -72 + 81$

$-18/2 = -9^2$

$x^2 - 18x + 81 = 9$

$(x-9)^2 = 9$

$x - 9 = 3 \quad x - 9 = -3$

$x = 12 \quad x = 6$

Ex3 $x^2 + 8x - 33 = 0$

$x^2 + 8x + 16 = 33 + 16$

$8/2 = 4^2$

$x^2 + 8x + 16 = 49$

$(x+4)^2 = 49$

$x + 4 = \pm 7$

$x + 4 = 7 \quad x + 4 = -7$

$x = 3 \quad x = -11$

Ex4 $x^2 + 16x - 9 = 0$

$x^2 + 16x + 64 = 9 + 64$

$x^2 + 16x + 64 = 73$

$(x+8)^2 = 73$

$x + 8 = \sqrt{73}$ or $x + 8 = -\sqrt{73}$

$x = \pm\sqrt{73} - 8$

$= 1.544$ or -16.544

Answer

IMPORTANT: When the quadratic you are solving has an 'a' term (the squared term) coefficient not equal to ___ it makes things a little more difficult. Do this extra step

Step 2 - The coefficient of the squared term must be divided out so that it is equal to one. *You cannot work with an a value that is greater than one*, Do not lose coefficient, carry it down.
Step 6- Multiply the right side of Step 5 by the coefficient you took out in Step 2

EXAMPLES:

Ex5. $2x^2 + 8x - 10 = 0$

$2x^2 + 8x + \_\_\_ = 10 + \_\_\_$

$2(x^2 + 4x + 4) = 10 + 2(4)$

$4/2 = 2^2$

$2(x^2 + 4x + 4) = 18$

$2(x+2)(x+2) = 18$

$2(x+2)^2 = 18$

$\sqrt{(x+2)^2} = \sqrt{9}$

$x + 2 = 3 \quad x + 2 = -3$

$x = 1 \quad x = -5$

Ex6. $3x^2 + 5x - 7 = 0$ ✱Tricky✱

$3x^2 + 5x + \_\_\_ = 7 + \_\_\_$

$3(x^2 + 5/3x + 25/36) = 7 + 3(25/36)$

$5/6^2$

$3(x + 5/6)(x + 5/6) = 109/12$

$3(x + 5/6)^2 = 109/12$

$(x + 5/6)^2 = 109/36$

$x + 5/6 = \sqrt{109/36}$

$\frac{5}{3} \cdot \frac{1}{2} = \frac{5}{6}$

$x = \pm\sqrt{109/36} - 5/6$

Solve each equation by completing the square.

1. $x^2-2x-15=0$

-3 & 5

$x^2-2x=15$

$x^2-2x+1=15+1$

$(x-1)^2=16$

$x-1=-4$ or $+4$

$x=-3$ or 5

2. $x^2+2x=35$

-7 & 5

3. $2x^2+8x-7=-2$

$\frac{-4\pm\sqrt{26}}{2}$

$2x^2+8x=5$

$2(x^2+4x+\underline{4})=5+2(\underline{4})$

$2(x+2)^2=13$

$(x+2)^2=13/2$

$x+2=\pm\sqrt{13/2}$

$x=\pm\sqrt{13/2}-2$

4. $8x=4x^2-1$

$\frac{2\pm\sqrt{5}}{2}$

$4x^2-1-8x=0$

$4x^2-8x=1$

$4(x^2-2x+\_\_)=1+4(\_)$

$4(x^2-2x+1)=1+4(1)$

$4(x-1)^2=5$

$(x-1)^2=5/4$

$x-1=\frac{\sqrt{5}}{2}$

$x=\pm\frac{\sqrt{5}}{2}+1$

same

same

5. $2x^2-4x+5=6$

$\frac{2\pm\sqrt{6}}{2}$

$2x^2-4x=1$

$2(x^2-2x+\underline{1})=1+2(\underline{1})$

$2(x-1)^2=3$

$(x-1)^2=3/2$

$x=\pm\sqrt{3/2}+1$

6. $6x=4x^2-1$

$\frac{3\pm\sqrt{13}}{4}$

7. $x^2+2x-8=0$

$-4,\ 2$

8. $x^2-7x=18$

$-2,\ 9$

$x^2-7x+\underline{(-7/2)^2}=18+\underline{(-7/2)^2}$

$x^2-7x+49/4=18+49/4$

$(x-7/2)(x-7/2)=121/4$

$(x-7/2)^2=121/4$

$\sqrt{(x-7/2)^2}=\sqrt{121/4}$

$x-7/2=\pm11/2$

$x=9$ or $x=-2$

9. $3x^2-2x-2=4$

$\frac{1\pm\sqrt{19}}{3}$

10. $-7x=3x^2-1$

$\frac{-7\pm\sqrt{61}}{6}$