

Complex Numbers- Combination of real and imaginary numbers



Note: Either part can be 0, so all Real Numbers and Imaginary Numbers are also Complex Numbers:

Complex Number	Real Part	Imaginary Part
$3 + 2i$	3	2
5	5	0
$-6i$	0	-6

Think back to Unit 1. If the discriminant of a quadratic is negative, there are no real solutions, but there are imaginary solutions.

Imaginary numbers happen when you try to take the square root of a negative number.

$$\sqrt{-1} = i$$

Which means $i^2 = -1$

Example: Can you simplify $\sqrt{-9}$?

Let's split the radical into 2 parts and try: $\sqrt{-9} = \sqrt{-1} * \sqrt{9} = i * 3 = 3i$

Rewrite the Following numbers using the imaginary numbers "i":

1. $\sqrt{-25}$ $5i$

2. $\sqrt{-81}$ $9i$

3. $\sqrt{-144}$ $12i$

4. $12 - \sqrt{-225}$ $12 - 15i$

5. $-48 + \sqrt{-256}$ $-48 + 16i$

6. Use what you know about complex numbers to find the imaginary solutions of $y = 5x^2 - 8x + 4$.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{8 \pm \sqrt{(-8)^2 - 4(5)(4)}}{2(5)} = \frac{8 \pm \sqrt{64 - 80}}{10} = \frac{8 \pm \sqrt{-16}}{10} = \frac{8 \pm 4i}{10}$$

$\frac{4 \pm 2i}{5}$

Now, let's combine what we know about Simplifying Radicals and Complex Numbers:

- | | | | |
|-----------------------------|---|--|---|
| 1. $\sqrt{-20}$ | $2i\sqrt{5}$ | 1). 20
$4^2 \cdot 5$
(2^2) | 2). 28
14^2
(7^2) |
| 2. $-\sqrt{-28}$ | $-2i\sqrt{7}$ | | |
| 3. $2\sqrt{-75}$ | $2 \cdot 5i\sqrt{3}$
$= 10i\sqrt{3}$ | 3). 75
$3^2 \cdot 25$
(5^2) | 4). 8
2^4
(2^2) |
| 4. $5\sqrt{-8}$ | $5 \cdot 2i\sqrt{2}$
$= 10i\sqrt{2}$ | | |
| 5. $\sqrt{-147}$ | $7i\sqrt{3}$ | 5). 147
$3^2 \cdot 49$
(7^2) | 6). 72
$9 \cdot 8$
$(3^2) \cdot (4^2)$
(2^2) |
| 6. $\sqrt{-72}$ | $2 \cdot 3i\sqrt{2} = 6i\sqrt{2}$ | | |
| 7. $\sqrt{-128}$ | $4 \cdot 2i\sqrt{2} = 8i\sqrt{2}$ | 7). 128
8^2
(8^2) | 8). Perfect square |
| 8. $\frac{3}{4}\sqrt{-144}$ | $\frac{3}{4} \cdot 12i$
$(9i)$ | 8). 16
4^2
(4^2) | |
| 9. $\sqrt{-512}$ | $4 \cdot 4i\sqrt{2}$
$= 16i\sqrt{2}$ | 9). 512
16^2
(16^2) | |

10. For i^x where x is an even number greater than zero, develop a rule to determine if i^x simplified is 1 or -1

If $x/2$ is an even number, then $i^x = 1$.

If $x/2$ is an odd number, then $i^x = -1$

Odd exponents (not divisible by 2) would be either i or $-i$

- $\sqrt{-1} = i$
- $-1 = i^2$
- $-i = i^3$
($-1 \cdot i$)
- $1 = i^4$
($-1 \cdot -1$)
- $i = i^5$
($-1 \cdot -i$)