Answer

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Solving Radical Equations: Practice Problems with Hints

Solve each of the following, and check your solutions.

1. $2\sqrt{x}-5=3$ (Hint: First solve for the $\sqrt{x}$.)

   $2\sqrt{x}=8$ $\sqrt{x}=4$ $x=16$

2. $x-2\sqrt{x}-15=0$ (Hint: Let $\sqrt{x}=n \Rightarrow x=n^2$, and then substitute.)

   $x-15=2\sqrt{x}$
   $(x-15)(x-15)=4x$
   $x^2-30x+225=4x$
   $x^2-34x+225=0$
   $x=25$ $x=9$

3. $\sqrt{x^2+2}=x+1$ (Hint: Don't forget, when you square a binomial, it becomes a trinomial.)

   $x^2+2=(x+1)(x+1)$
   $x^2+2=x^2+2x+1$
   $1=2x$
   $1/2=x$

4. $\sqrt{\dfrac{3x+1}{x-1}}=\sqrt{2x-1}$ (Hint: Make sure you check for extraneous solutions.)

   $\frac{3(3)+1}{3-1}=\frac{9+10}{2}=\sqrt{19}=\sqrt{5}$
   $\sqrt{-1}=\sqrt{-1}$
   $x=0$ & $x=3$
   $\frac{3x+1}{x-1}=2x-1$
   $3x+1=(2x-1)(x-1)$
   $3x+1=2x^2-3x+1$
   $0=2x^2-6x$
   $2x(x-3)$
   $2x=0$ $x-3=0$
   $x=0$ $x=3$

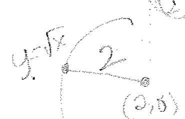
5. $x^{\frac{1}{2}}=x-2$ (Hint: Remember what $x^{\frac{1}{2}}$ equals.)

   $\sqrt{x}=x-2$ $x=(x-2)(x-2)$
   $x=x^2-4x+4$
   $0=x^2-5x+4$
   4 1

Challenge Problems

6. The distance from a point on the curve $y=\sqrt{x}$ to the point (2,0) is equal to 2 at what points on the curve? (Hint: Draw a picture.)

   LAST

   $\sqrt{(\Delta x)^2+(\Delta y)^2}=2$

7. Finish the right side of the equation $\sqrt{3x-5}=\sqrt[3]{ax}$ by finding a so that $x = 5$ is a solution to the equation. (Hint: If $x = 5$ satisfies the equation, what does that mean?)

   ≈ 3.16
   $\sqrt{3(5)-5}=\sqrt{10}\approx 3.16$
   $\sqrt[3]{ax}\approx 3.16$
   $ax\approx 31.62$
   $\frac{31.62}{5}$
   $a\approx 6.32$

8. Solve $\left(\sqrt{2x}+\sqrt{x+1}\right)^2=\left(\sqrt{x+41}\right)^2$ (Hint: Don't forget, when you square a binomial, it becomes a trinomial.)

   Binomial
   $(\sqrt{2x}+\sqrt{x+1})(\sqrt{2x}+\sqrt{x+1})=x+41$
   $2x+\sqrt{2x^2+2x}+\sqrt{2x^2+2x}+x+1=x+41$
   $3x+1+2\sqrt{2x^2+2x}=x+41$
   $2\sqrt{2x^2+2x}=-2x+40$
   $\sqrt{2x^2+2x}=x+20$

9. Solve $\sqrt{2x-5}=3-x$, and verify your solution, using a graphing calculator. (Hint: You will need to use the quadratic formula.)

   $2x-5=(3-x)(3-x)$
   $2x-5=9-6x+x^2$
   $0=x^2-8x+14$
   can't factor
   2.6 & 5.6

10. Solve $x^{\frac{2}{3}}-x^{\frac{1}{3}}-2=0$. (Hint: Let $u=x^{\frac{1}{3}}$, and solve for u first.)

   $u^2-u-2=0$
   $(u-2)(u+1)$
   $u=2$ $u=-1$
   $2=x^{1/3}$ $-1=x^{1/3}$
   $2=\sqrt[3]{x}$ $-1=\sqrt[3]{x}$
   $8=x$ $-1=x$
   Extraneous (Both $=-2$)

Practice: Solving Radical Equations Same Base

1) $4^{2x+3}=1$

   $-3/2$
   $4^{2x+3}=4^0$
   $2x+3=0$
   $2x=-3$
   $x=-3/2$
   $\log_4 1$

2) $5^{3-2x}=5^{-x}$

   3
   $3-2x=-x$
   $3=x$

3) $3^{1-2x}=243$

   $3^{1-2x}=3^5$
   -2
   $1-2x=5$
   $-2x=4$
   $x=-2$

4) $3^{2a}=3^{-a}$

   0
   $2a=-a$
   $3a=0$
   $a=0$

5) $4^{-2x}\cdot 4^{x}=64$

   4^3
   -3
   $-2x+x=3$
   $-x=3$
   $x=-3$

6) $6^{-2x}\cdot 6^{-x}=\dfrac{1}{216}$

   1
   $6^{-2x}\cdot 6^{-x}=6^{-3}$
   $-2x-x=-3$
   $-3x=-3$
   $x=1$
   $\log_6 216$
   $\log(216)/\log(6)$
   $=3$
   6^3

7) $2^{x}\cdot\dfrac{1}{32}=32$

   10
   $2^x\cdot 2^{-5}=2^5$
   $x-5=5$
   $x=10$

8) $2^{-3p}\cdot 2^{2p}=2^{2p}$

   0
   $-3p+2p=2p$
   $-p=2p$
   $0=3p$

9) $64\cdot 16^{-3x}=16^{3x-2}$

   $7/12$
   $4^3\cdot(4^2)^{-3x}=(4^2)^{3x-2}$
   $3+-6x=6x-4$
   $12x=7$
   $x=7/12$

10) $\dfrac{81^{3n+2}}{243^{-n}}=3^4$

   $-4/17$
   $81^{3n+2}\,243^{n}=3^4$
   $(3^4)^{3n+2}(3^5)^n=3^4$
   $12n+8+5n=4$
   $17n+8=4$
   $17n=-4$

- Application -

1.) The voltage V of an audio system's speaker can be represented by $V = 4\sqrt{p}$, where P is the power of the speaker. An engineer wants to design a speaker with 400 watts of power. What will the voltage be?

$V = 4\sqrt{p}$
$V = 4\sqrt{400}$ → $V = 4 \cdot 20$
$= 80$

2.) The velocity v of an object dropped from a tall building is given by the formula $v = \sqrt{64d}$ where d is the distance the object has dropped. Solve the formula for d.

$v = \sqrt{64d}$
$v^2 = 64d$
$d = \frac{v^2}{64}$

3.) The radius in inches of a balloon can be expressed as $r = \sqrt[3]{\frac{3V}{4\pi}}$; where r is the radius and V is the volume of the balloon in cubic inches. If air is pumped to inflate the balloon from 500 cubic inches to 800 cubic inches, by how many has the radius increased?

a. What was the radius of the balloon originally? $r = \sqrt[3]{\frac{3(500)}{4\pi}} = \sqrt[3]{119} \approx 5$
b. What was the radius after inflating the balloon to 800 cubic inches? $r = \sqrt[3]{\frac{3(800)}{4\pi}} = 5.75$
c. How can you use the 2 radii to find the amount of increase?

Subtract Big - Small
$\approx .75$ inches

4.) Boat builders share an old rule of thumb for sail boats. The maximum speed K in knots is about 1.35 times the square root of the length L in feet of the boats waterline.

a. Write an equation to describe the relationship between speed and length of the waterline. $K = 1.35\sqrt{L}$
b. A customer is planning to order a sailboat with a maximum speed of 12 knots. How long should the waterline be? $12 = 1.35\sqrt{L}$ → $8.88 = \sqrt{L}$
$\approx 3 = L$
c. How much longer would the waterline have to be to achieve a maximum speed of 15 knots?
$15 = 1.35\sqrt{L}$
$11.11 = \sqrt{L}$
About 1/3 ft longer

5.) The formula $t = \sqrt{\frac{2s}{a}}$ shows the time t that any vehicle takes to travel a distance s at a constant acceleration a, starting from rest. What is the difference in time between a car accelerating at $16m/s^2$ and one accelerating at $25m/s^2$ for a distance of 200 m?

a. What is the time that a car accelerating at $16m/s^2$ takes to travel 200 m? $\sqrt{\frac{2(200)}{16}} = 5$ sec.
b. What is the time that a car accelerating at $25m/s^2$ takes to travel 200 m? $\sqrt{\frac{2(200)}{25}} = 4$ sec
c. What is the difference between the 2 accelerations?

1 second