Key

Guided Notes (Unit 3): Properties of Exponents

If $a, b, x, y \in \Re$ and $a, b, \neq 0$, and $m, n \in Z$ then the following properties hold:

1. **Negative Exponent Rule:** $b^{-n} = \frac{1}{b^n}$ and $\frac{1}{b^{-n}} = b^n$ **Answers must never contain negative exponents.**

Examples:

a) 5^{-3} $\quad \frac{1}{5^3} = \frac{1}{125}$

b) $\frac{1}{4^{-2}}$ $\quad \frac{4^2}{1} = 16$

2. **Zero Exponent Rule:** $b^0 = 1$

Examples:

a) 7^0 $\quad 1$

b) $(-5)^0$ $\quad 1$

c) -5^0 $\quad -1$ Be careful! Look at parenthesis

3. **Product Rule:** $b^m \cdot b^n = b^{m+n}$ - keep the base and add the exponents

Examples:

a) $2^2 \cdot 2^3$ $\quad 2^5 = 32$

b) $x^{-3} \cdot x^7$ $\quad x^4$

4. **Quotient Rule:** $\frac{b^m}{b^n} = b^{m-n}$ - keep the base and subtract the exponents.

Examples:

a) $\frac{2^{12}}{2^4}$ $\quad 2^8 = 256$

b) $\frac{x^2}{x^5}$ $\quad x^{-3} = \frac{1}{x^3}$

5. **Power to a Power Rule** $(b^m)^n = b^{m \cdot n}$ - keep the base and multiply the exponents.

Examples:

a) $(2^2)^5$ $\quad 2^{10} = 1,024$

b) $(x^{-3})^4$ $\quad x^{-12} = \frac{1}{x^{12}}$

6. **Product to a power:** $(ab)^n = a^n b^n$ - raise both bases to the same power.

Note well: $(a+b)^n \neq a^n + b^n$ and $(a-b)^n \neq a^n - b^n$

Examples:

a) $(-2y)^4$ $\quad (-2)^4 y^4 \rightarrow 16y^4$

b) $(3x^{-2}y^5)^2$ $\quad 3^2 x^{-4} y^{10} \rightarrow \frac{9y^{10}}{x^4}$

7. **Quotient to a power:** $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ - raise both bases to the same power.

Examples:

a) $\left(\frac{2}{5y}\right)^4$ $\quad \frac{2^4}{5^4 y^4} = \frac{16}{625y^4}$

b) $\left(\frac{x^{-3}}{3y^{-2}}\right)^2$ $\quad \frac{x^{-6}}{3^2 y^{-4}} = \frac{y^4}{9x^6}$

Simplifying Exponents

Step	Method	Example
1	Label all unlabeled exponents "1"	$\left(\frac{25x^{-6}y(z^{-11})^2}{5(x^{-2})^5y^8z^2}\right)^{-2}$
2	Take the reciprocal of the fraction and make the outside exponent positive.	$\left(\frac{5(x^{-2})^5y^8z^2}{25x^{-6}y(z^{-11})^2}\right)^{2}$
3	Get rid of any inside parentheses.	$\left(\frac{5x^{-10}y^8z^2}{25x^{-6}yz^{-22}}\right)^{2}$
4	Reduce any fractional coefficients.	$\left(\frac{1x^{-10}y^8z^2}{5x^{-6}yz^{-22}}\right)^{2}$
5	Move all negatives either up or down. Make the exponents positive.	$\left(\frac{1y^8z^2z^{22}x^6}{5x^{10}y}\right)^{2}$
6	Combine all like bases.	$\left(\frac{1y^7z^{24}}{5x^4}\right)^{2}$
7	Distribute the power to all exponents.	$\frac{1y^{14}z^{48}}{25x^8}$ All done !!

Simplifying Exponential Expressions

1) No parentheses.
2) No powers raised to a powers.
3) Each based occurs only once.
4) No negative exponents.
5) Simplify numerical expressions.

good checklist

Examples: Simplify the expression.

a) $(2x^8y^{-4})^2$ → $2^2x^{16}y^{-8}$ → $\frac{4x^{16}}{y^8}$

b) $(-3x^4y^5)^3$ → $-3^3x^{12}y^{15}$ → $-27x^{12}y^{15}$ (negative bases don't move)

c) $(-7xy^4)(-2x^5y^6)$ → $14x^6y^{10}$

d) $(-6x^2y^5)(3xy^{-3})$ → $-18x^3y^{-15}$ → $\frac{-18x^3}{y^{15}}$

e) $\frac{-35x^2y^4}{5x^6y^{-8}}$ → $-7x^{-4}y^{12} = \frac{-7y^{12}}{x^4}$

f) $\frac{100x^{-2}y^6}{20x^{-4}y^3}$ → $5x^2y^3$ (Be careful subtracting negatives)

g) $\left(\frac{4x^2d^3}{y^{-7}g^4}\right)^{-3}$ → $\frac{4^{-3}x^{-6}d^{-9}}{y^{21}g^{-12}}$ → $\frac{1g^{12}}{64x^6d^9y^{21}} = \frac{1}{64}$

h) $\left(\frac{5x^{-8}}{2y^4}\right)^{-5}$ (switch first is an easy option) → $\left(\frac{2y^4}{5x^{-8}}\right)^5$ → $\frac{2^5y^{20}}{5^5x^{-40}}$ → $\frac{32y^{20}x^{40}}{3125}$