Unit 1 Review

Function Notation

A function is a mathematical relation so that every X (input) in the domain corresponds with one Y (output) in the range. To evaluate a function, f(x), substitute the X value for every x and calculate.

Example: Evaluate f(-3) for $f(x) = 100(2)^x$.

$f(-3) = 100(2)^{-3}$

$f(-3) = 12.5$

Transformations

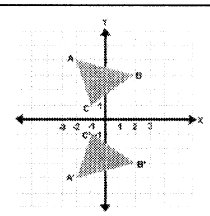
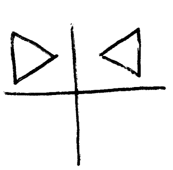
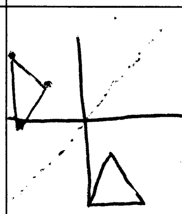
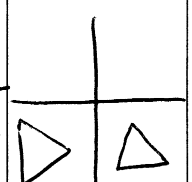
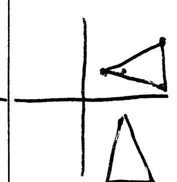
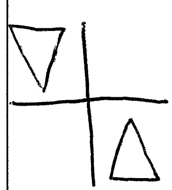
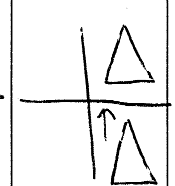
Transformations are function rules that applied to points of the coordinate plane to create a new shape.

Certain transformation preserve rigid motion and produce congruent figures: translations, reflections, and rotations, or any combination of these.

Other transformations do not preserve rigid motion, so they do not produce congruent figures. dilations produce similar figures, while stretches and shrinks are not congruent or similar.

To prove if a transformation preserves rigid motion, you can use the distance formula: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Rules for transformations:

Transform-ation	Reflection Over the x-axis	Reflection Over the y-axis	Reflection Over the y=x line	Rotation of 90° Clockwise	Rotation of 90° Counter-clockwise	Rotation of 180°	Trans-lation
Written Description	Shape flips over the x-axis (flips over the horizontal axis)	Shape flips over y-axis (flips over vertical axis)	Shape flips over y=x (diagonal) line	Shape turns 90° right around origin ($\frac{1}{4}$ around the plane)	Shape turns 90° left around origin ($\frac{1}{4}$ around the plane)	Shape turns 180° (halfway) around origin	Shape Slides
Picture							
Function Rule	$f(x, y) \rightarrow (x, -y)$	$f(x,y) \rightarrow (-x,y)$	$f(x,y) \rightarrow (y,x)$	$f(x,y) \rightarrow (y,-x)$	$f(x,y) \rightarrow (-y,x)$	$f(x,y) \rightarrow (-x,-y)$	$f(x,y) \rightarrow (x \pm h, y \pm k)$

To determine the coordinates for a dilation, Multiply each point times the scale factor of the dilation.

Concept Questions:

1. Why do rotations, reflections, and translations preserve congruence while dilations do not?

Dilations make a shape bigger or smaller because points are multiplied, while the others just move the shape around.

2. Why do adding and subtracting translate points, while multiplying dilates points?

Adding and subtracting moves the coordinates' locations, while dilation (multiplying) changes the points' location in relation to each other.

Unit 2 Review

Polynomial Operations

Multiplying: Distribute terms times EVERY other term

To distribute exponents, write the polynomial in parentheses and multiply out.

Adding or subtracting: combine like terms.

Remember, you can NOT operate with variables in the calculator!

Example 1: $(2x-3)^3$

$(2x-3)(2x-3)(2x-3)$ Write out binomials

$(4x^2-6x-6x+9)(2x-3)$ Distribute first two binomials

$(4x^2-12x+9)(2x-3)$ Combine like terms

$8x^3-12x^2-24x^2+36x+18x-27$ Distribute last binomial

$\boxed{8x^3-36x^2+54x-27}$

Factoring/Dividing

GCF

$10x^2-5x$

5 divides into 10 and 5

x divides into x^2 and x

$\boxed{5x(2x-1)}$

x^2+bx+c

$x^2-9x-22$

$\underline{-11}+\underline{2}=-9$

$\underline{-11}\cdot\underline{2}=-22$

$\boxed{(x-11)(x+2)}$

ax^2+bx+c

$3x^2-13x-10$

$\underline{-15}+\underline{2}=-13$

$\underline{-15}\cdot\underline{2}=-30$

$(3x^2-15x)+(2x-10)$

$3x(x-5)+2(x-5)$

$\boxed{(3x-2)(x-5)}$

Perfect Squares

x^2-49

$\sqrt{x^2}=x$

$\sqrt{49}=7$

$\boxed{(x+7)(x-7)}$

$5x^3+500x$

$\boxed{5x(x^2+100)}$

Cannot factor with real numbers

Quadratic Formula

You MUST use the quadratic formula for imaginary/complex solutions or irrational solutions in radical form.

Example: Solve $3x^2+9x=-11$

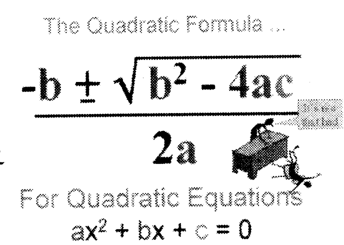
Set Equal to 0 $3x^2+9x+11=0$

$a=3$ $b=9$ $c=11$

$$\frac{-9\pm\sqrt{81-132}}{6}=\frac{-9\pm\sqrt{-51}}{6}$$

$$\boxed{\frac{-9\pm i\sqrt{51}}{6}}$$

Negative inside radical → pull out i, imaginary number $\sqrt{-1}$



Completing the Square

To complete the square and rewrite quadratics, use $\left(\frac{b}{2}\right)^2$ to find the correct c.

Then, add inside the parentheses and subtract outside the parentheses.

Finally, factor and combine.

Example: Complete the square to find the vertex of $y = x^2 - 12x - 15$. Then, solve the equation.

Complete the Square

$\left(\frac{b}{2}\right)^2=\left(\frac{-12}{2}\right)^2=36$ $\quad y=(x^2-12x+36)-15-36$

$y=(x-6)^2-51$

Solve

$0=(x-6)^2-51$

$51=(x-6)^2$

$x-6=\sqrt{51}$ $\quad x-6=-\sqrt{51}$

$\boxed{x=6\pm\sqrt{51}}$

Solving Equations/Systems

Solutions to all equations and systems are the points of intersection on the graph.

If you graph both sides of an equation (or both equations in a system) in the calculator, use:

2^{nd} - Trace (Calc), #5 Intersect to find the solution. (Don't forget to adjust your window range if necessary.)

Real-World Quadratics

x-intercept: Where y-value = 0

y-intercept: Starting value, where X = 0

Maximum/minimum value: The highest or lowest y-coordinate (output)

Example: A rocket is launched and follows the function $h(t) = -16t^2 + 500t + 30$ for its first 10 seconds.

a) From what height is the rocket launched?

Y starting

y-intercept, where x = 0

$y = -16(0)^2 + 500(0) + 30 = \boxed{30}$

b) What is the highest height the rocket reaches?

Maximum Y

Vertex - $x = \frac{-B}{2a} = \frac{-500}{-32} = 15.625$

$y = -16(15.625)^2 + 500(15.625) + 30 = \boxed{7592.5}$

OR GRAPH - 2^{nd} Trace (Calc) - #4 Maximum

c) When does the rocket hit the ground?

X Y/height = 0

x-intercept, solve equation or

GRAPH - 2^{nd} Trace (Calc) - #2 Zero $\boxed{x = 31.31}$

Concept Questions:

1. Why is a parabola shaped like a U, and why does it have a line of symmetry through the vertex?

Squaring a number makes it positive, so the negative x-values become positive y-values at the same value as when x is positive (so the line of symmetry exists).

2. What is the easiest way to solve quadratics? Explain.

Answers will vary.

3. Why is $x^2 - 49$ not equal to $(x - 7)(x - 7)$?

Distributing $(x-7)(x-7)$ yields two $-7x$ terms, which combine to $-14x$, not 0.
Distributing $(x-7)(x+7)$ yields $-7x + 7x$, which combines to 0, so there is no middle term.

Unit 3 Review

Simplifying Radicals

To simplify a number or expression under a square root, determine the largest perfect square factor of the radicand under the radical, write the expression by multiplying the two radicals, and take the square root of the perfect square. For example:

$\sqrt{72} = \sqrt{36} \bullet \sqrt{2} = 6\sqrt{2}$ Examples: $\sqrt{150}$

$\sqrt{25} \cdot \sqrt{6}$

$\boxed{5\sqrt{6}}$

$\sqrt{60x^8}$

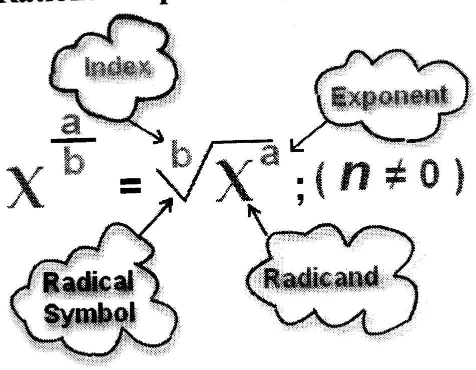
$\sqrt{60}$ $\sqrt{x^8}$

$\sqrt{4} \cdot \sqrt{15}$ x^4

$2\sqrt{15}$

$\boxed{2x^4\sqrt{15}}$

Rational Exponents (with fractions)



The numerator of the exponent is the exponent of the radicand.

The denominator of the exponent is the root index.

Radical Equations

To solve a radical equation (with a variable inside a radical), first use inverse operations to get the radical by itself.

Then, square both sides to pull the variable out of the radical.

Finally, solve the equation to get the variable by itself, if necessary.

Example: $5\sqrt{3x+2} + 19 = 99$

$-19 \quad -19$

$\frac{5\sqrt{3x+2}}{5} = \frac{80}{5}$

$(\sqrt{3x+2})^2 = (16)^2$

$3x + 2 = 256$

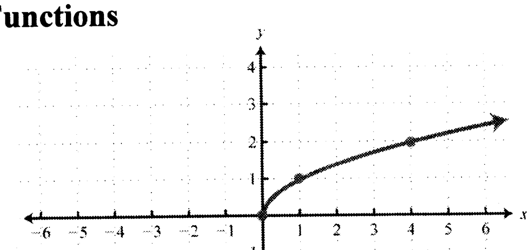
$3x = 254$

$\boxed{x = \frac{254}{3} \approx 84.667}$

Radical Functions

$f(x) = \sqrt{x}$

x	$f(x)$
0	0
1	1



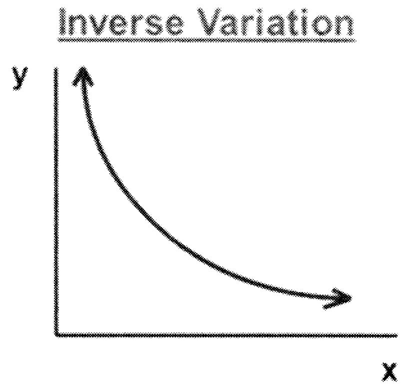
Domain - $x \geq 0$ Range - $y \geq 0$

x-intercept - $x = 0$ y-intercept - $y = 0$

minimum point - (0, 0)

All in Quadrant I

Inverse Variation



Inverse variation results when two variables are multiplied to equal a constant, *k*.

The relationship is that as one variable increases, the other variable decreases.

x and **y** vary inversely if for a constant **k**

$xy = k$ or $y = \frac{k}{x}, k \neq 0$

Concept Questions:

1. Why are the domain and range of the parent radical function non-negative numbers?

Domain - There are no square roots in the real number system for negative numbers.
Range - to keep it a function, only positive square roots are included.

2. Using rational exponents, explain why a square root and an exponent of 2 are inverse operations.

Square Root - $\sqrt{x} = x^{\frac{1}{2}}$ $(x^{\frac{1}{2}})^2 = x^{\frac{1}{2}\cdot 2} = x$, so they end at the same value ("cancel" each other out)

Exponent 2 - x^2

3. What are the main differences between direct and inverse variation?

Direct - As one number increases, other increases Linear

Inverse - As one number increases, other decreases Non-linear

Unit 4 Review

Triangle Congruence

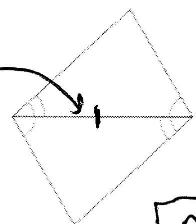
Two triangles are congruent when all Sides and all angles are congruent.

We can prove that two triangles are congruent if we know that certain parts of the triangles are congruent by proving congruence postulates: SSS, SAS, ASA, AAS, HL (for right triangles)

Examples:

1.

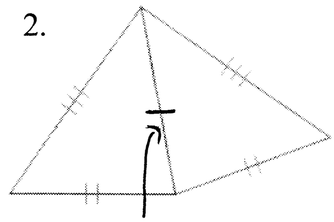
Reflexive Side



$\boxed{\text{ASA}}$

2 Angles, 1 Side,
Side included between angles

2.

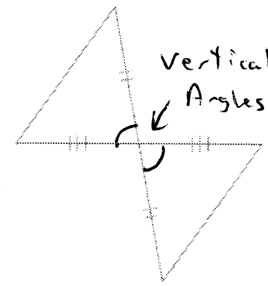


Reflexive Side

3 Sides $\boxed{\text{SSS}}$

3.

Vertical Angles



2 Sides, 1 Angle,
Angle Included Between Sides

$\boxed{\text{SAS}}$

Similar Triangles

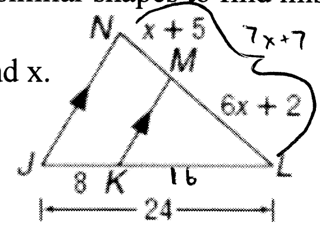
ANY shapes are similar if their sides are proportional.

If you divide the length of the corresponding sides, the ratios should be equal. The ratio is called the Scale factor.

Triangles are similar if all angles (or any two) are equal. This is the AA similarity postulate.

You can use similar shapes to find missing lengths of sides.

Example: Find x.



$\frac{16}{24} = \frac{6x+2}{7x+7}$

$16(7x+7) = 24(6x+2)$

$112x + 112 = 144x + 48$

$112 = 32x + 48$

$64 = 32x$

$\boxed{2 = x}$

Other Geometric Theorems

The <u>midsegment</u> of a triangle is parallel and half the length of the opposite side.

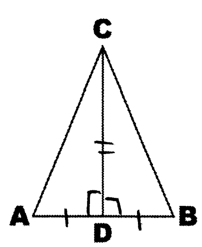
Side-Splitter Theorem - Any segment in a triangle parallel to a side divides the sides into proportional parts.

All angles in a triangle add to 180°, and isosceles triangles have 2 equal angles and sides.

We need to know these theorems, but we also need to be able to PROVE these theorems.

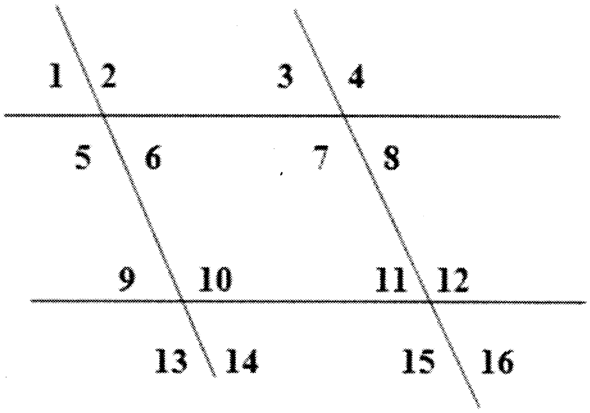
Given: CD is the perpendicular bisector of AB.

Prove: ΔABC is isosceles.



We know ∡APC ≅ ∡BDC = 90° because CD and AB are perpendicular, and we know AD ≅ BD because CD is the bisector. We know CD ≅ CD by the reflexive property, so ΔADC ≅ ΔBDC by SAS Congruence. So AC ≅ BC by CPCTC, and ΔABC is isosceles.

Theorems about angles:



Equal Angles		Supplementary Angles	
Vertical Angles	∡4 and ∡7	**Linear Pair**	∡5 and ∡6
Corresponding Angles	∡9 and ∡11	**Consecutive Interior Angles**	∡6 and ∡10
Alternate Interior Angles	∡7 and ∡12		
Alternate Exterior Angles	∡16 and ∡9		

Other Answers Work Too - these are Examples!

Concept Questions:

1. What are the similarities and differences between similar and congruent triangles?

Lots of answers work, but the most important is:

Congruent - Equal Angles, Equal Sides Similar - Equal Angles, Proportional Sides

2. In your own words, what does it mean to "prove" that two triangles are congruent using one of the congruence postulates?

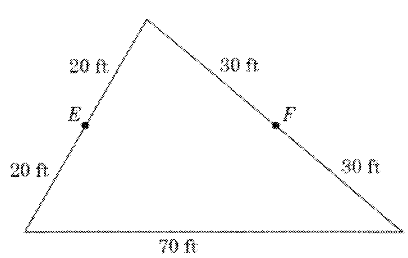
It means that all parts are congruent if we can prove certain ones are congruent by a postulate.

3. What is the scale factor of the similar triangles created by the midsegment of a triangle? How do you know?

2, because it is half the length of the opposite side

Two More Sample Problems!

How long is $\overline{EF}$?



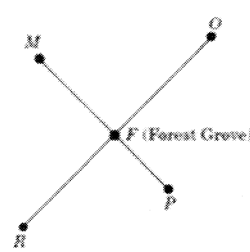
A 20 ft

B 25 ft

C 30 ft

D 35 ft

According to the map, the road connecting the cities of Oakton (O) and Ridgeton (R) intersects the road connecting Maple View (M) and Pineville (P).



If the roads intersect in the town of Forest Grove (F) in the diagram, which statement is *always* true?

A $MP \cong RO$

B $\overline{PF} \perp \overline{OF}$

C $\angle OFP \cong \angle RFM$

D $\angle RFP \cong \angle MFR$

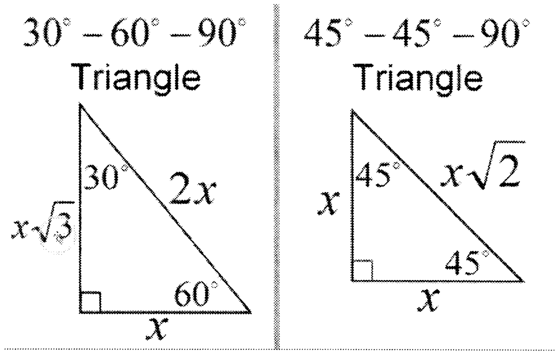
Unit 5 Review

Pythagorean Theorem

$Leg^2 + Leg^2 = Hypotenuse^2$

Don't forget to Simplify the radical if necessary for your answer.

Special Right Triangles (45-45-90 and 30-60-90)



The altitude of an equilateral triangle forms two 30-60-90 triangles

The diagonal of a square forms two 45-45-90 isosceles triangles

Trigonometric Ratios (MAKE SURE YOU ARE IN DEGREE MODE IN YOUR CALCULATOR!!!)

Sin = $\dfrac{\text{opposite leg}}{\text{hypotenuse}}$ Cos = $\dfrac{\text{adjacent leg}}{\text{hypotenuse}}$ Tan = $\dfrac{\text{opposite leg}}{\text{adjacent leg}}$

The three trigonometric ratios apply to the angles. The side lengths can be any size, but the ratios will hold for that angle measure.

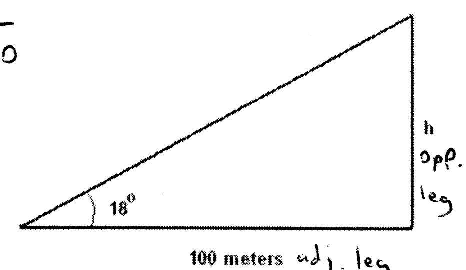
To set up problems to solve for the length of a side:

1. Determine the angle and sides you are working with
2. Determine the appropriate trigonometric ratio
3. Solve to isolate the variable

$\tan 18 = \dfrac{h}{100}$

$100 \tan 18 = h$

$\boxed{32.49m = h}$



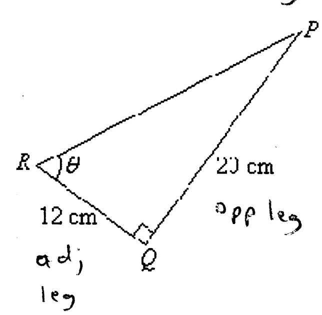
To set up problems to solve for an angle measure:

1. Determine the angles and sides you are working with
2. Determine the appropriate trig ratio
3. Use the inverse trig ratio ($\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$)

$\tan \theta = \dfrac{20}{12}$

$\tan^{-1}\left(\dfrac{20}{12}\right) = \theta$

$\boxed{59.04^\circ = \theta}$



Concept Questions:

1. Why do trig ratios hold for angles when the side lengths can be any length?

All ~~angles~~ triangles with the same angle measures are similar, so the ratios of their sides are equal.

2. How do special right triangle rules relate to the Pythagorean Theorem?

The rules follow the Pythagorean Theorem: $(x)^2 + (x\sqrt{3})^2 = (2x)^2$ and $(x)^2 + (x)^2 = (x\sqrt{2})^2$

Unit 6 Review

Probability Concepts

Probability - A value between 0 and 1 that determines the likelihood of a specific event occurring

Experimental Probability - The actual probability that occurs from an experiment or simulation

Theoretical Probability - The expected probability based on the mathematical likelihood of an event occurring

The more outcomes/trials that occur for a given experiment, the closer the experimental probability will be to the theoretical probability.

Probability Terms

Independent Events - Events whose outcomes are not determined by other or previous events

Dependent Events - Events whose likelihood depends by other events

Mutually Exclusive - Events or outcomes that cannot both happen

Conditional Probability - When the likelihood of an event is dependent on another event occurring

(represented as $B \mid A$, or B given A)

Probability Formulas

Addition Rule (Mutually Exclusive Events) - When two events, A and B, are mutually exclusive, the probability that A or B will occur is the sum of the probability of each event. $P(A \text{ or } B) = P(A) + P(B)$

Addition Rule (Non-Mutually Exclusive Events) - When two events, A and B, are non-mutually exclusive, the probability that A or B will occur is: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Multiplication Rule (Independent Events) - When two events, A and B, are independent, the probability of both occurring is: $P(A \text{ and } B) = P(A) \cdot P(B)$

Multiplication Rule (Dependent Events) - When two events, A and B, are dependent, the probability of both occurring is: $P(A \text{ and } B) = P(A) \cdot P(B|A)$

Concept Questions:

1. What are the differences between the addition and multiplication rules, and when would each apply?

Addition - When more than one outcome is calculated out of the whole

Multiplication - When the probability is calculated for multiple outcomes at once

2 Why does experimental probability get closer to theoretical probability as the number of events increases?

The chance of random variation decreases as the number of trials increases.