

Solving Radical Equations

Key

Pg# _____

A "radical" equation is an equation in which at least one **variable** expression is stuck inside a radical, usually a square root.

For instance, this is a radical equation: $\sqrt{x} + 2 = 6$

...but this is not: $x + \sqrt{2} = 6$

To Solve a Radical Equation:

1. Isolate the sq root (the variable) on one side of the equation
2. Square both sides of the equation

Key note: You must square both sides, not *individual terms* on each side

Example 1:

$$(\sqrt{x^2 + 8})^2 = (2\sqrt{2x-1})^2$$

$$x^2 + 8 = 4(2x-1)$$

$$x^2 + 8 = 8x - 4$$

$$x^2 - 8x + 12 = 0$$

$$\begin{array}{r} 12 \\ -6 \quad -2 \\ \hline -8 \end{array}$$

$$x-6=0 \quad x-2=0$$

$$\boxed{x=6} \quad \checkmark \quad \boxed{x=2} \quad \checkmark$$

Plug in to check

Example 2:

$$\sqrt{x} - 4 = \sqrt{9x}$$

$$-4 = \sqrt{9x} - \sqrt{x}$$

$$-4 = 3\sqrt{x} - \sqrt{x}$$

$$(-4)^2 = (2\sqrt{x})^2$$

$$16 = 4 \cdot x$$

$$16 = 4x$$

$$4 = x$$

plug in to check

$$\sqrt{4} - 4 = \sqrt{9 \cdot 4}$$

$$2 - 4 \neq 6$$

Example 3:

$$(x-1)^2 = \sqrt{5x-9}^2$$

$$(x-1)(x-1) = 5x-9$$

$$x^2 - 2x + 1 = 5x - 9$$

$$x^2 - 7x + 10 = 0$$

$$x-5=0 \quad x-2=0$$

$$\boxed{x=5} \quad \checkmark \quad \boxed{x=2} \quad \checkmark$$

$$\begin{array}{r} 10 \\ -5 \quad -2 \\ \hline -7 \end{array}$$

plug in to check

Extraneous

Extraneous Solutions:

An Extraneous Solution is a **solution** of a simplified version of an equation that does not satisfy the original equation.

What does that mean??

Example:

Solve: $(\sqrt{x+4})^2 = (x-2)^2$

$$x+4 = (x-2)(x-2)$$

$$x+4 = x^2 - 4x + 4$$

$$0 = x^2 - 5x$$

$$x(x-5)$$

$$x=0$$

$$x-5=0$$

$$x=5$$

Your answers should be $x=5$ and $x=0$

When you check your answers back into the original equation, do they both work?

Let $x=5$: $\sqrt{5+4} = 5-2$
 $3=3$

Let $x=0$: $\sqrt{0+4} = 0-2$
 $2 \neq -2$

So only $x=5$ is a real solution to this problem (and 0 is extraneous).

Therefore, we must check carefully our answers to watch for extraneous solutions!

PRACTICE:

1) $4\sqrt{y} - 6 = 18$

$y=36$

2) $\sqrt{3x} + 2 = 5$

$x=3$

3) $3\sqrt{x-7} + 5 = 20$

$x=32$

4) $\sqrt{2x} + 4 = 8$

$x=6$

5) $3\sqrt{x} + 8 = 12$

$x=16/9$

6) $7 + \sqrt{1-5x} = 16$

$x=-16$

7) $\sqrt{x-15}^2 = (3-\sqrt{x})^2$

EXTRANEUS

$x-15 = (3-\sqrt{x})(3-\sqrt{x})$

$x-15 = 9 - 3\sqrt{x} - 3\sqrt{x} + x$

$x-15 = 9 - 6\sqrt{x} + x$

$-24 = -6\sqrt{x}$
 $4 = \sqrt{x}$
 $x=16$

8) $4 + \sqrt{x-1} = x-3$

$\sqrt{x-1} = x-7$

$x-1 = (x-7)(x-7)$

$x-1 = x^2 - 14x + 49$

$0 = x^2 - 15x + 50$

$x=10$ $x=5$

9) $\sqrt{\frac{2x}{3} + 1} = \sqrt{5}$

$x=6$

10) Solve: $\sqrt{2x+1} - \sqrt{x} = 1$

$\sqrt{2x+1} = 1 + \sqrt{x}$

$2x+1 = (1+\sqrt{x})(1+\sqrt{x})$

$2x+1 = 1 + 2\sqrt{x} + x$

$x^2 = (2\sqrt{x})^2$

$x^2 = 4x$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x=0$ $x=4$

extraneous

BOTH work