Unit 3 – Rational & Radical Functions

Exponent Rules

63. $x^4 * x^6 =$ $x^{4+6} = x^{10}$
64. $(y^7)^2 =$ $y^{7\cdot 2} = y^{14}$
65. What is the perimeter of a square with a side length of $2x^3$? (add) $2x^3 + 2x^3 + 2x^3 + 2x^3 = 8x^3$
66. $\frac{z^2}{z^4} =$ $z^{-2} = 1/z^2$
67. $\frac{x^{(y+2)}}{x^{(y+1)}} =$ $x^{y+2-y+1} = x^1 = x$
68. $(\frac{2d}{d^2})^3 =$ $\frac{2^3 d^3}{d^6} = 8d^{3-6} = \frac{8}{d^3}$

Radical & Rational Exponents If the expression is written in radical form, rewrite it in fractional exponent form. If the expression is in fractional exponent form, rewrite it in radical form.

69. $d^{\frac{1}{3}} =$ $\sqrt[3]{d}$
70. $(4x^3y^4)^{\frac{3}{4}} =$ $\sqrt[4]{(4x^3y^4)^3} = \sqrt[4]{64x^9y^{12}}$
71. $(6x^2y)^{\frac{2}{3}} =$ $\sqrt[3]{(6x^2y)^2}$
72. $\sqrt[3]{2y} =$ $(2y)^{1/3}$
73. $-\sqrt[4]{(3x)^6} =$ $-(3x)^{6/4} = -(3x)^{3/2}$
74. $\sqrt[2]{(3x^2y)^4} =$ $(3x^2y)^{4/2} = (3x^2y)^2 = 9x^4y^2$ (most simplified)

Simplify Radicals

75. $\sqrt{16} =$ 4
76. $\sqrt{\frac{175}{7}} =$ $\sqrt{25} = 5$
77. $5\sqrt{180} =$ $5\cdot 2\cdot 3\sqrt{5} = 30\sqrt{5}$
78. $-2\sqrt{72} =$ $-2\cdot 2\cdot 3\sqrt{2} = -12\sqrt{2}$
79. $\sqrt{6300} =$ $2\cdot 3\cdot 5\sqrt{7} = 30\sqrt{7}$
80. $7\sqrt{96} =$ $2\cdot 2\sqrt{2\cdot 3} = 4\sqrt{6}$

Add & Subtract Radicals Remember: You can only combine like radicals, just like when you combine like terms!!!!

81. $\sqrt{x} + 2\sqrt{x} + 3\sqrt{x} =$ $6\sqrt{x}$
82. $3\sqrt{7} - 5\sqrt{7} =$ $-2\sqrt{7}$
83. $\sqrt{6} + \sqrt{6} =$ $2\sqrt{6}$
84. $\sqrt{x} + 4\sqrt{y} - 4\sqrt{x} - 2\sqrt{y} =$ $-3\sqrt{x} + 2\sqrt{y}$
85. $4\sqrt{32} + 3\sqrt{18} - \sqrt{8} =$ $23\sqrt{2}$
86. $6\sqrt[3]{4} - 6\sqrt{2} + 3\sqrt[3]{4} - 3\sqrt{2} =$ $9\sqrt[3]{4} - 9\sqrt{2}$

Multiply Radicals Remember: multiply what is in front of the radicals, multiply what's inside the radicals (they turn in to one radical), then simplify the radical.

87. $\sqrt{2} * \sqrt{6} =$ $\sqrt{12} = 2\sqrt{3}$
88. $2\sqrt{3} * 4\sqrt{12} =$ $8\sqrt{36} = 8\cdot 6 = 48$

89. $4z^3\sqrt{8x^5y} * 2x\sqrt{3z^4} =$ $16x^3z^5\sqrt{6xy}$

90. $\sqrt{3}(\sqrt{x} - \sqrt{y}) =$ $\sqrt{3x} - \sqrt{3y}$

91. $5\sqrt{5}(\sqrt{3} + 4) + \sqrt{3}(6 - \sqrt{5}) =$ $4\sqrt{15} + 20\sqrt{5} + 6\sqrt{3}$

FOIL 92. $(6 + \sqrt{x})(2 - \sqrt{x}) =$ $12 - 4\sqrt{x} - x$

93. $4(4 + 2\sqrt{x})(\sqrt{x} - 4) =$ $-16\sqrt{x} + 8x - 64$

distribute then FOIL

Rationalize the Denominator

94. $\frac{\sqrt{5}}{\sqrt{3y}} = \frac{\cdot\sqrt{3y}}{\cdot\sqrt{3y}} = \frac{\sqrt{15y}}{3y}$

95. $\frac{x-3}{\sqrt{x-4}} = \frac{\cdot\sqrt{x-4}}{\cdot\sqrt{x-4}} = \frac{(x-3)(\sqrt{x-4})}{x-4}$

Divide Radicals

96. $\frac{\sqrt{90}}{3\sqrt{26}} = \frac{3\sqrt{10}}{3\sqrt{26}} = \frac{\sqrt{65}}{13}$

97. $\frac{\sqrt{5}}{5\sqrt{7}} =$ $\frac{\sqrt{35}}{35}$

98. $\frac{\sqrt{20xy}}{3\sqrt{5xy^5}} =$ $\frac{2}{3y^2}$

99. $\frac{\sqrt{7x^5y^3}}{4\sqrt{3xy^5}} =$ $\frac{x^2\sqrt{21}}{12y}$

Solving Radical Equations

100. $(\sqrt{x+3})^2 = 4^2$ $= x+3=16$ $= x=13$

101. $\sqrt{4x-2} - 3 = 4x$ no solutions (imaginary)

102. $\sqrt{a^2-4} = 4a + 1$ ↓

103. $\sqrt{x-2} = 3x$

104. $\sqrt[3]{(x-2)^3} + 4 = 20$ $x=18$

Dont forget to look for extraneous solutions

Direct Variation

$y = kx$

Y varies directly with x.

Example: Y varies directly with x. If $y = 4$ and $x = 2$, find x when $y = 6$.

$y = kx$	$y = kx$
$4 = k(2)$	$6 = (2)x$
$2 = k$	$3 = x$

*You try these:

105. Is this an example of direct variation? If so, write the equation to represent this data. $y=kx$

x	y
-2	-6
-1	-3
0	0
1	3
2	6

$-6 = k(-2)$ $k=3$

$-3 = k(-1)$ $k=3$

yes, direct

$y=3x$

106. Y varies directly with x. If $y = 16$ and $x = 8$, find x when $y = 4$. $y=kx$ $\boxed{x=2}$

107. Y varies directly with x. If $y = 10$ and $x = 2$, find y when $x = 3$.

$y=kx$

$\boxed{y=15}$

Inverse Variation

$$y = \frac{k}{x}$$

Y varies inversely with x.

Example: Y varies inversely with x. If $y = 4$ and $x = 2$, find x when $y = 6$.

$$y = \frac{k}{x} \qquad y = \frac{k}{x}$$

$$4 = \frac{k}{2} \qquad 6 = \frac{8}{x}$$

$$8 = k \qquad 6x = 8$$

$$x = \frac{8}{6} = \frac{4}{3}$$

*You try these:

108. Is this an example of inverse variation? If so, write the equation to represent this data. $y = \frac{k}{x}$ or $xy=k$

x	y
-2	-6
-1	-12
1	12
2	6
3	4

$-2 \cdot -6 = 12$

$-1 \cdot -12 = 12$

$1 \cdot 12 = 12$

$2 \cdot 6 = 12$

$3 \cdot 4 = 12$

so, yes, inverse

$k=12$

$y = 12/x$

109. Y varies inversely with x. If $y = 16$ and $x = 8$, find x when $y = 4$. $y=k/x$ $\boxed{x=32}$

110. Y varies inversely with x. If $y = 10$ and $x = 2$, find y when $x = 3$. $\boxed{y = 20/3}$

111. The number of hours needed to paint a house varies inversely with the number of painters working. A 2400 square foot house can be painted in 27 hours by 6 painters. How many painters would it take to paint the house in 18 hours?

$h = k/p$ $\qquad 27 = k/6$ $\qquad k=162$ $\qquad h = \frac{162}{p}$ plug in $\qquad \boxed{p=9}$

REMEMBER: If the problem says directly or jointly, you are going to multiply the variable by k. If the problem says inversely, you are going to divide by the variable!!!!!

Joint Variation

$$y = kxz$$

Y varies jointly with x and z.

Example: y varies jointly with x and z. If $y = 18$, $x = 2$, and $z = 3$, find y when $x = 4$ and $z = 2$

$$18 = k(2)(3) \qquad y = (3)(4)(2)$$

$$18 = k(6) \qquad y = 24$$

$$3 = k$$

*You try these:

112. y varies jointly with x^2 and a^2. If $y = 8$, $x = 3$, and $a = 4$, find y when $x = 4$ and $a = 2$ $\boxed{y = 32/9}$

113. y varies jointly with x and z. If $y = 8$, $x = 2$, and $z = 3$, find y when $x = 3$ and $z = 4$

$8 = k(2)(3)$

$k = 4/3$ $\rightarrow$ $y = 4/3\, xz$ then plug in

$\boxed{y=16}$

Combined Variation

$$y = \frac{kxz}{w}$$

Y varies jointly with x and z and inversely with w.

Example: Y varies jointly with x and z and inversely with w. If $y = 3$, $x = 2$, $z = 3$ and $w = 4$, find y when $x = 4$, $z = 3$ and $w = 5$

$$y = \frac{kxz}{w} \qquad\qquad y = \frac{kxz}{w}$$

$$3 = \frac{k(2)(3)}{4} \qquad\qquad y = \frac{(2)(4)(3)}{5}$$

$$12 = k(6) \qquad\qquad y = \frac{24}{5}$$

$$2 = k \qquad\qquad y = 4.8$$

*You try these:

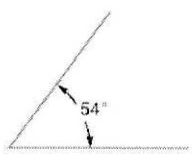
114. Y varies jointly with x and z and inversely with w. If $y = 5$, $x = 2$, $z = 3$ and $w = 1$, find y when $x = 3$, $z = 2$ and $w = 3$ $\boxed{y = 5/3}$

115. Y varies jointly with x and z and inversely with w. If $y = 5$, $x = 4$, $z = 3$ and $w = 2$, find z when $y = 4$, $x = 3$ and $w = 6$ $\boxed{z = 48/5}$

Unit 4 – Similarity & Congruence

Classifying Triangles

- Classify by the Angles
  - Acute



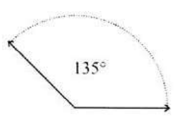
Example: A triangle can be any of the following combinations:

Scalene Acute — Equilateral Acute
Scalene Right — Equilateral Right
Scalene Obtuse — Equilateral Obtuse
Isosceles Acute
Isosceles Right
Isosceles Obtuse

*Name the triangle by its sides and by its angles.

  - Right

  - Obtuse



- Classify by the Sides
  - Scalene

  - Isosceles