

Topic: Writing Equations from Roots

Objective: to be able to create a quadratic equation that represents the information given

Steps to write an equation in Standard Form:

(think of this as the same process we use when solving quadratics, but you are working backwards)

1. Pull needed information from problem. For these problems, you will need the two roots and another point on the parabola. If something isn't given to you, find it.
2. Set roots equal to 0.
3. Set factors equal to 0.
4. Simplify by multiplying the binomials.
5. Check to see if there is a Greatest Common Factor.
6. If you found a GCF, distribute it to your equation.
7. Graph the equation on the calculator and check the table to make sure the given points are there. This will prove that your answer is correct.

Example with two roots and a point given:

Write the equation of a quadratic with the zeros 4 and -2 that includes the point (2, -24).

$$x = 4 \qquad x = -2$$

$$x - 4 = 0 \qquad x + 2 = 0$$

$$(x - 4)(x + 2) = 0$$

$$x^2 - 4x + 2x - 8 = 0$$

$$x^2 - 2x - 8 = 0$$

$$y = a(x^2 - 2x - 8)$$

$$-24 = a((2)^2 - 2(2) - 8)$$

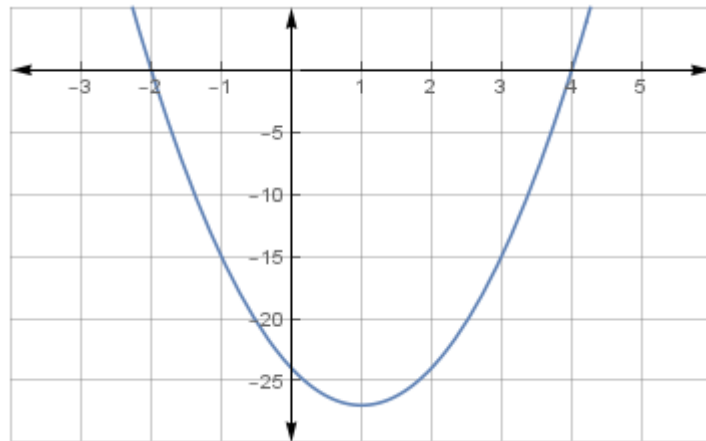
$$-24 = a(4 - 4 - 8)$$

$$-24 = a(-8)$$

$$3 = a$$

$$y = 3(x^2 - 2x - 8)$$

$$y = 3x^2 - 6x - 24$$



Example with a graph given:

Write a quadratic equation that represents the graph.

$$x = -5 \quad x = -1 \quad \text{and a point: } (-3, 8)$$

$$x + 5 = 0 \quad x + 1 = 0$$

$$(x + 5)(x + 1) = 0$$

$$x^2 + 5x + x + 5 = 0$$

$$x^2 + 6x + 5 = 0$$

$$y = a(x^2 + 6x + 5)$$

$$8 = a((-3)^2 + 6(-3) + 5)$$

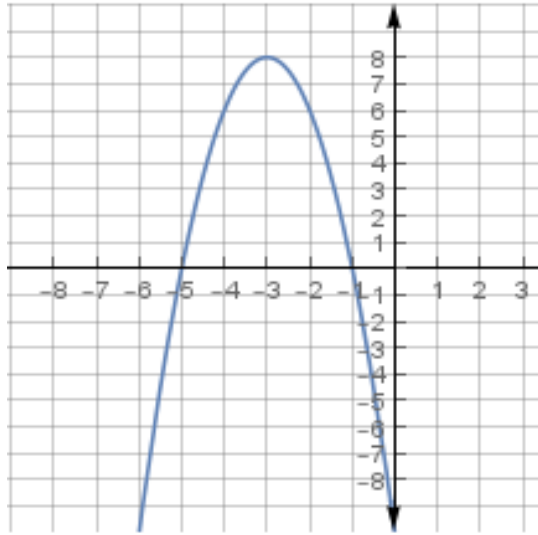
$$8 = a(9 - 18 + 5)$$

$$8 = a(-4)$$

$$-2 = a$$

$$y = -2(x^2 + 6x + 5)$$

$$y = -2x^2 - 12x - 10$$

**Example with two zeros and the maximum value:**

Write a quadratic equation given a zero at 1 and 7 and a maximum value of 27.

$$x = 1 \quad x = 7 \quad \text{vertex: } (4, 27)$$

$$x - 1 = 0 \quad x - 7 = 0$$

$$(x - 1)(x - 7) = 0$$

$$x^2 - x - 7x + 7 = 0$$

$$x^2 - 8x + 7 = 0$$

$$y = a(x^2 - 8x + 7)$$

$$27 = a((4)^2 - 8(4) + 7)$$

$$27 = a(16 - 32 + 7)$$

$$27 = a(-9)$$

$$-3 = a$$

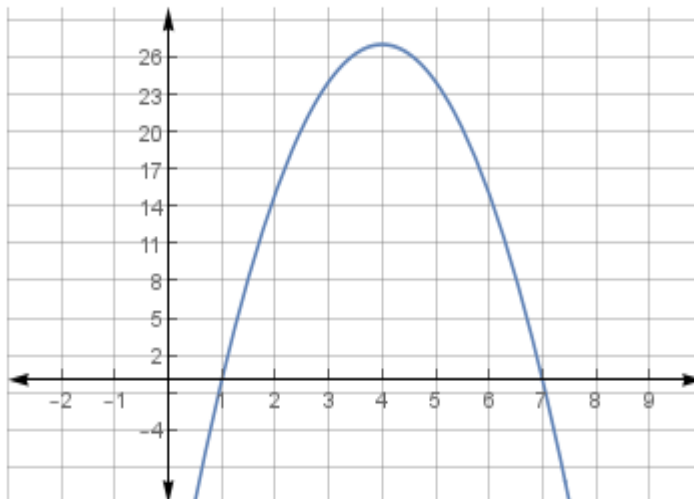
$$y = -3(x^2 - 8x + 7)$$

$$y = -3x^2 + 24x - 21$$

Side Note:

The maximum value is the largest y-value.

The minimum value is the smallest y-value.



Example with one zero and the minimum point:

Write the equation of a quadratic with an x-intercept at $x = -3$ and a minimum at $(0, -9)$.

$$x = -3 \quad x = 3 \quad \text{vertex: } (0, -9)$$

$$x + 3 = 0 \quad x - 3 = 0$$

$$(x + 3)(x - 3) = 0$$

$$x^2 + 3x - 3x - 9 = 0$$

$$x^2 - 9 = 0$$

$$y = a(x^2 - 9)$$

$$-9 = a((0)^2 - 9)$$

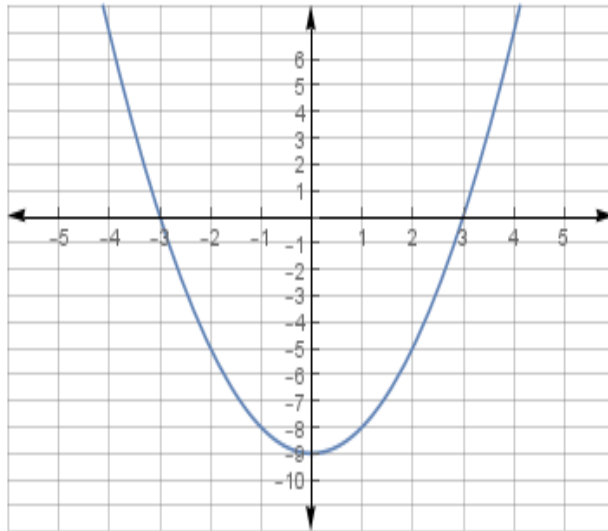
$$-9 = a(0 - 9)$$

$$-9 = a(-9)$$

$$1 = a$$

$$y = 1(x^2 - 9)$$

$$y = x^2 - 9$$



Practice:

1. Write the equation of a quadratic with the zeros at -3 and 4 that includes the point $(-4, 16)$.

$$y = 2x^2 - 2x - 24$$

2. Write a quadratic equation with roots $(-2, 0)$ and $(4, 0)$ that has a minimum value of -36 .

$$\text{Missing value: } (1, -36) \text{ to get } y = 4x^2 - 8x - 32$$

3. Write a quadratic equation with a solution at $x = -5$ and a maximum at $(-3, 4)$.

$$\text{Missing zero: } x = -1 \text{ to get } y = -x^2 - 6x - 5$$

4. Write an equation that represents the quadratic on the graph.

I used the point $(-4, 20)$ to get $a = 20/21$ for $y = 20/21(x^2 + 18x + 77)$ <<- distribute that fraction out

5. If you draw a picture of the arch as a parabola starting at the Origin (0, 0), and label the points based on this information, the points would be:
- (0, 0)
 - (12, 0)
 - (2, 8) or (10, 8)

Once you have these points, you can find your equation: $y = -2/5x^2 + 24/5x$